

Large-scale Cosmic Homogeneity from a Multifractal Analysis of the PSCz Catalogue

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ABSTRACT

We investigate the behaviour of galaxy clustering on large scales using the PSCz catalogue. In particular, we ask whether there is any evidence of large-scale fractal behaviour in this catalogue. We find the correlation dimension in this survey varies with scale, consistent with other analyses. For example, our results on small and intermediate scales are consistent those obtained from the QDOT sample, but the larger PSCz sample allows us to extend the analysis out to much larger scales. We find firm evidence that the sample becomes homogeneous at large scales; the correlation dimension of the sample is $D_2 = 2.992 \pm 0.003$ for $r > 30h^{-1}$ Mpc. This provides strong evidence in favour of a Universe which obeys the Cosmological Principle.

Key words: Cosmology: theory – large-scale structure of the Universe – Methods: statistical

1 INTRODUCTION

The most fundamental assumption underlying the standard cosmological models is that, on sufficiently large scales, the Universe is homogeneous and isotropic. We know that, on small scales, matter is distributed in a roughly hierarchical fashion with galaxies being grouped into clusters which in turn are grouped into superclusters. This has led some to argue for a radically different cosmological paradigm in which this hierarchy carries on *ad infinitum* (Coleman, Pietronero & Sanders 1988; Coleman & Pietronero 1992). In such a case the Universe is not smooth on large scales, but has a fractal structure in which the Cosmological Principle, at least in its usual form, does not apply (Sylos-Labini et al. 1998). On the other hand, many others have argued that redshift surveys show a definite transition to large-scale homogeneity (e.g. Guzzo 1997; Cappi et al. 1998; Martínez et al. 1998; Scaramella et al. 1998).

There is still some degree of controversy about the large-scale homogeneity of the Universe, primarily because the main sources of direct information on spatial structure, redshift surveys, have either been too shallow or too sparse or of too restricted a geometry to provide conclusive evidence. This has led to arguments about the treatment of boundary effects and other sampling errors, leading to different groups reporting different results for the same surveys (e.g. Martínez & Jones 1990; Sylos-Labini et al. 1998). Given these bones of contention, the best arguments in favour of large-scale homogeneity still stem from the near-isotropy of

sources or background radiation observed in projection on the sky (Wu, Lahav & Rees 1999).

In this paper we address this issue by performing a multifractal analysis of the PSCz catalogue of redshifts of IRAS galaxies. This is a good sample to use for this issue because it is both extremely deep, allowing truly large-scale structure to be probed but is also all-sky (reducing the influence of boundary corrections). The method we use involves a generalization of the fractal dimension to a set of generalised dimensions based on the scaling properties of different moments of the galaxy counts (Martínez et al. 1995). The work we describe here represents the culmination of work begun by Martínez & Coles (1994) which analyzed the QDOT catalogue, a one-in-six subsample of the parent IRAS catalogue (Lawrence et al. 1999).

2 THE SAMPLE

The PSCz catalogue consists of 15411 IRAS galaxies across 84% of the sky with flux at $60\mu\text{m}$ greater than 0.60 Jy. In all, the redshifts of 14677 galaxies are available (Saunders et al 2000). We use a subset of this catalogue, defined as follows. First, we exclude galaxies with galactic latitudes $|b| < 10^\circ$ for this analysis to avoid problems near the galactic plane. In order to minimize the influence of local structure and large-scale sampling problem, the distance is limited in the range $10h^{-1} \text{ Mpc} < R < 400h^{-1} \text{ Mpc}$, where R is calculated from

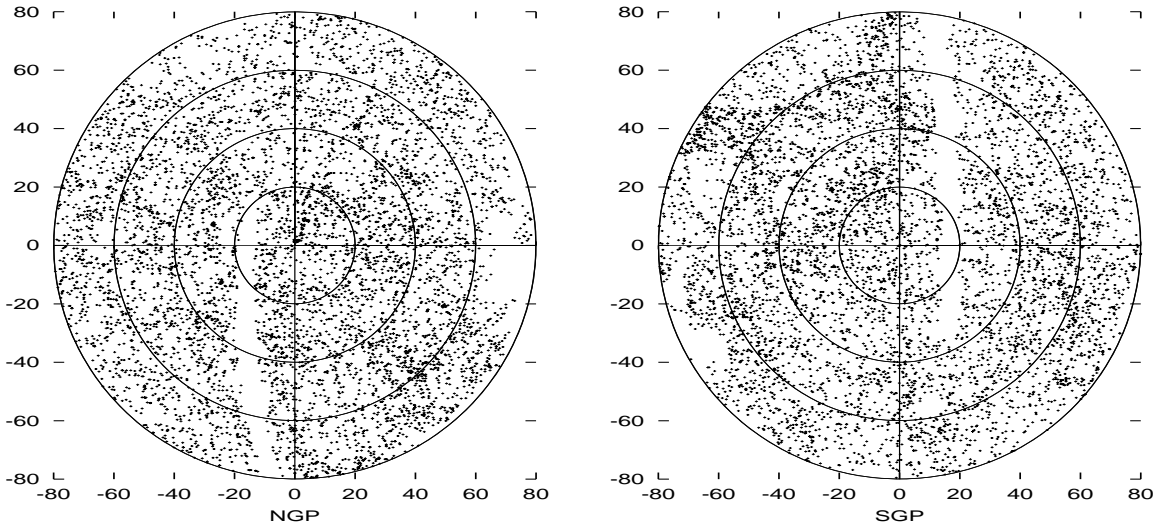


Figure 1. The projection of galaxies selected for the sample in the northern and southern hemispheres, respectively.

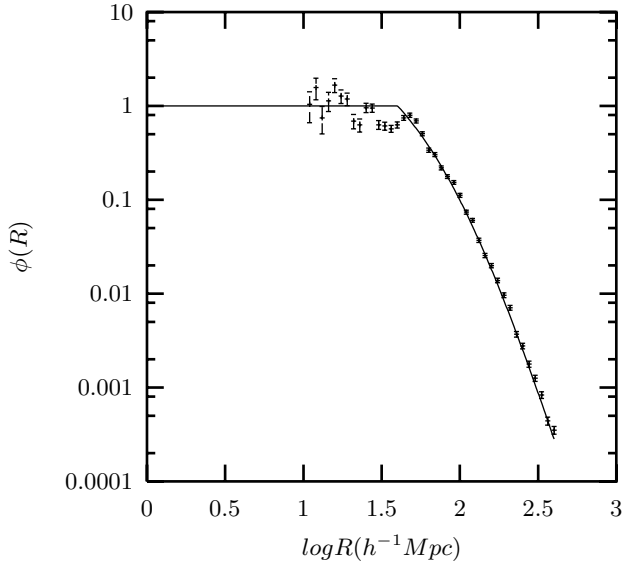


Figure 2. Selection Function. The solid line is the expected function while points with error bars are the actual counting result of the sample.

the redshift z via the Mattig formula:

$$R = \frac{c}{H_0 q_0^2 (1+z)} [q_0 z + (q_0 - 1)(\sqrt{2q_0 z + 1} - 1)], \quad (1)$$

with $H_0 = 100h^{-1} \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0.5$.

As usual, we define the selection function $\phi(R)$ as the probability that a galaxy at a distance R is included in the catalogue. An cutoff in absolute luminosity is set in order that $\phi(R) = 1$ when $R < 40h^{-1} \text{ Mpc}$. For a 60-micron flux limit of $S_{60} > 0.60$ the luminosity cutoff is $10^{9.19} L_\odot$ at $40h^{-1}$. The form of $\phi(R)$ can be derived directly from the parametric selection function given by Saunders et al (2000). The final sample has 11901 galaxies. For reference the distribution of galaxies is shown in Figure 1, while Figure 2 is the plot of $\phi(R)$ against R .

3 MULTIFRACTAL ANALYSIS

The measure we use is constructed from the partition function,

$$Z(q, r) = \frac{1}{N} \sum_{i=1}^N p_i(r)^{q-1} \propto r^{\tau(q)}, \quad (2)$$

with $p(i) = n_i(r)/N$, where $n_i(r)$ is the count of objects in the cell of radius r centered upon an object labeled by i (which is not included in the count). For each value of q in equation (2), one can have a different scaling exponent of the set $\tau(q)$.

It is necessary in practical applications such as this to account for edge effects and selection. We take the corrected local count around the i th object to be

$$n_i(r) = \frac{1}{f_i(r)} \sum_{j=1}^N \frac{\Psi(|\mathbf{r}_j - \mathbf{r}_i| - r)}{\phi(r_j)}, \quad (3)$$

and

$$\Psi(x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}, \quad (4)$$

where $f_i(r)$ is the volume fraction of the sphere centered on the object of radius r within the boundary of the sample.

The scaling exponents $\tau(q)$ lead to the definition of the so-called Renyi dimensions:

$$D_q = \frac{\tau(q)}{q-1}, \quad (5)$$

where D_q is the spectrum of fractal dimensions for a fractal measure on the sample. The D_q for each value of q gives information about the scaling properties of different aspects of the density field. For high q , D_q tells us about high-density regions while for low q (including negative values) the measure is weighted towards low-density regions. For $q = 1$, D_1 can be derived from

$$S(r) = \frac{1}{N} \sum_{i=1}^N \log p_i(r) \propto r^{D_1}, \quad (6)$$

where $S(r)$ is the partition *entropy* of the measure on the sample set; D_1 is consequently termed the information dimension.

Of more direct interest in this case is the case $q = 2$. The exponent D_2 is what is generally called the correlation dimension, and it is related to the usual two-point correlation function $\xi(r)$ for a sample displaying large-scale homogeneity (Peebles 1980). If the mean number of neighbours around a given point is $\langle n \rangle$ then

$$\langle n \rangle = 4\pi\bar{n} \int_0^r [1 + \xi(r)] s^2 ds. \quad (7)$$

In this case $\langle n \rangle \sim r^\alpha$ means $\alpha = D_2$. A homogeneous distribution has $D_2 = 3$, whereas a power-law in $1 + \xi(r) \sim r^{-\gamma}$ yields $D_2 = 3 - \gamma$.

4 RESULTS AND DISCUSSION

We illustrate the results in Figure 3 by plotting D_q against cell size R for $q = 2$. Considering the case $q = 2$ first, it is interesting to compare the results with those obtained for the QDOT sample (Martínez & Coles 1994). For r below $\sim 10h^{-1}$ Mpc, we get $D_2 = 2.16$. The QDOT value was 2.25. Above $r \sim 30h^{-1}$ Mpc, $D_2 = 2.99$ which closely approaches the value $D = 3$ for homogeneous distribution. The formal error for the fit is around 0.003, of similar size to the Poisson error.

Within the range $10h^{-1}$ Mpc to $30h^{-1}$ Mpc, there is an intermediate regime represented by a gradual transitions from fractal to homogeneous behaviour. To compare with the range quoted by Martínez & Coles (1994), we fit this intermediate regime ($10h^{-1}$ Mpc $< r < 50h^{-1}$ Mpc) and obtained $D_2 = 2.71$, consistent with the value $D_2 = 2.77$ obtained for the QDOT catalogue. Notice that, on small scales, the value of D_2 will be affected by peculiar motions since we work entirely in redshift space, but this is not expected to be the case on large scales.

We show the results for $q = 5$ in order to illustrate some of the difficulties with estimating D_q for large q . Notice that for scales larger than $\sim 50h^{-1}$ Mpc, $\tau(q) = 11.8$, so $D_5 = 2.95$. This is consistent with the tendency to homogeneity discussed in the previous paragraph. However, on smaller scales (say below $r \sim 20h^{-1}$ Mpc), the partition function displays a series of steps as a function of r . This systematic tendency is clearer in the PSCz sample than in QDOT and is due to the suppression of high q to the low value of $n_i(r)$ at certain levels. The end points of the steps probably correspond to some characteristic scales of the hierarchical structures at different scales. A possible *primary* scaling law could be recovered by only adopting the end points of each step. This fitting shows that $D_5 = 1.23$. Between $20h^{-1}$ and $50h^{-1}$ Mpc, $D_5 = 2.05$ in comparison with the value of 2.30 obtained for QDOT.

The spectrum D_q for $q > 2$ is shown in Figure 4. The measure of Equation (2) is not capable of generating a complete spectrum including $q < 2$ for a point set because of discreteness effects. In order to cover the latter range, an alternative measure is used:

$$W[\tau(q), n] = \frac{1}{N} \sum_{i=1}^N r_i(n)^{-\tau} \propto n^{1-q}, \quad (8)$$

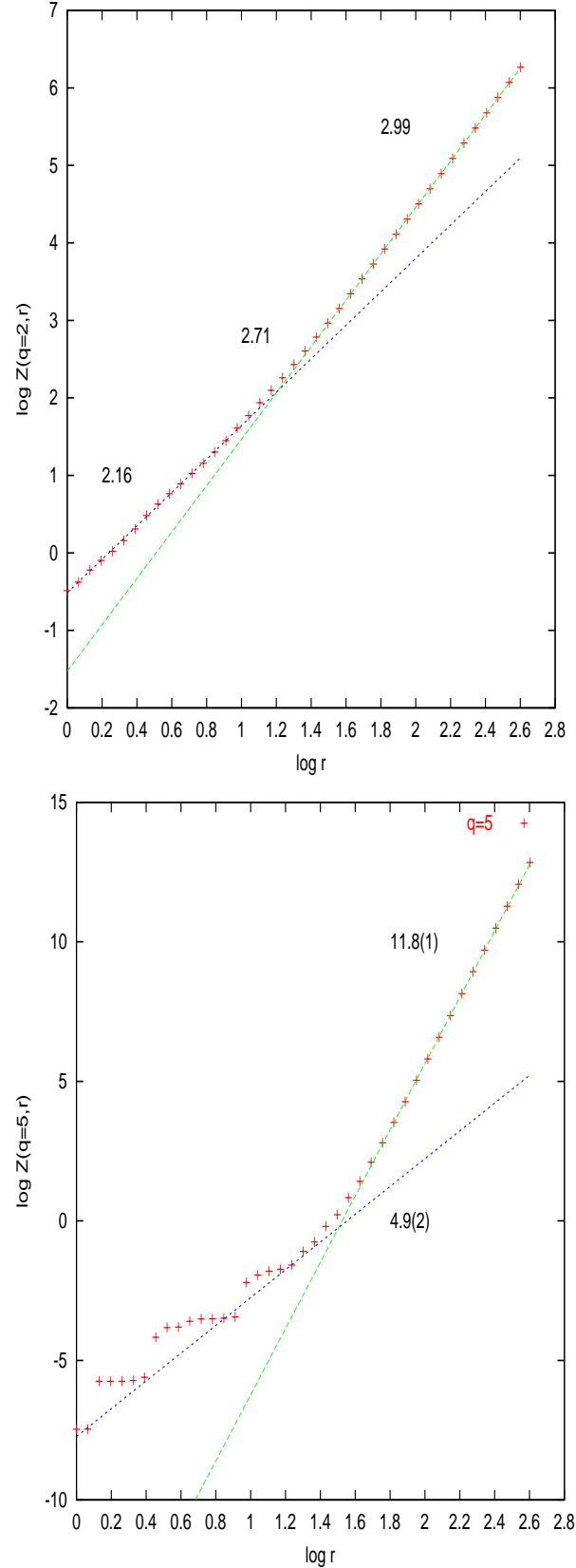


Figure 3. $\log Z(q, r)$ vs. $\log r$ for $q = 2$ and $q = 5$. The fit for intermediate scales is not plotted.

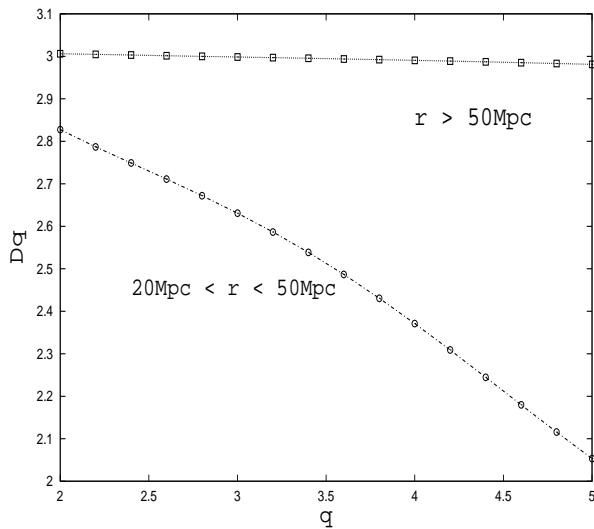


Figure 4. The generalized dimensions D_q ($q > 2$) for $r > 50h^{-1}$ Mpc and r between 20 and $50h^{-1}$ Mpc respectively.

where $r_i(n)$ is the radius of the smallest sphere centered at an object that encloses n neighbors. We shall return to the case $q < 2$ in future work.

5 CONCLUSIONS

The results we have presented here provide very strong evidence that the distribution of IRAS galaxies becomes homogeneous on large scales. This is reassuring for adherents of the standard cosmological framework but runs counter to the alternative, fractal paradigm.

Our future work on this problem will follow two principal directions. One is to investigate thoroughly the role of boundary corrections and sampling in the estimation of D_q , including $q < 2$, using simulated catalogues. Our work along these lines so far has shown that the large-scale value of D_2 is robust to changes in boundary correction, at least for this catalogue, so we can be confident about its extremely small error bar.

The other important thread is to connect the scaling behaviour of the matter distribution, in terms of D_q , to the gravitational dynamics of structure formation. Simple scaling arguments have already borne considerable fruit in the interpretation of large-scale clustering (e.g. Hamilton et al. 1991). With the imminent arrival of even larger surveys, the use of more sophisticated descriptors will allow a deeper understanding of how spatial pattern arises in the galaxy distribution.

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